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Binary, Decimal, Hexadecimal

Why Do We Normally Use Decimal?

The simple explanation to why we use decimal numbers is that we have two hands, each containing five dactyl appendages, making ten total dactyl appendages. Because the cultures that use base ten are in power today, everyone seems to be using it for most purposes. This convention of using decimal numbers is extremely useful toward helping students who still count on their fingers and toes excel in mathematics.

Why Do Computers Use Binary?

Computers use binary numbers because their memory cells store only two values, "1" and "0" (also known as "on" and "off", or "true" and "false"). Their lowest level components also only operate on these two values. This means its simplest form of knowledge is as simple as "yes" and "no". People who count using their fingers have ten digits and computers counting on their transistors' "on" and "off" states have two digits.

Why Do Computer Scientists Use Hexadecimal?

After awhile, bigger numbers in binary can start to take up large quantities of space (and become harder to type). Because hexadecimal is base sixteen, its quantity of digits is a power of two. This makes it reasonably simple to convert back and forth between base sixteen and base two. One digit in base sixteen represents four digits in base two, so we only use a quarter of the space displaying numbers in hexadecimal as we would in binary. (Not forgetting to mention the health bonuses of reduced typing.)

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The Bases Compared

The values of a digit in each base are represented by specially selected characters. Binary has two digits, so it's standard to use '0' and '1'. Decimal has ten digits, so it uses '0', '1', '2', '3', '4', '5', '6', '7', '8', and '9'. Hexadecimal has sixteen digits, so it uses '0', '1', '2', '3', '4', '5', '6', '7', '8', '9', 'A', 'B', 'C', 'D', 'E', and 'F'. The values of these digits are compared in the chart below:

Binary	Decimal	Hexadecimal
0	0	0
1	1	1
10	2	2
11	3	3
100	4	4
101	5	5
110	6	6
111	7	7
1000	8	8
1001	9	9
1010	10	A
1011	11	B
1100	12	C
1101	13	D
1110	14	E
1111	15	F
10000	16	10

The way we know what value a number has is by taking the individual digits and their position in the number and applying them to a special equation dependent on each number system. The equations have the form:

$$\sum_{n=0}^{\infty}(d_n \cdot b^n)+\sum_{m=0}^{-\infty}(d_m \cdot b^m)$$

n = The digit position after the decimal point going from right to left.
m = The digit position before the decimal point going from left to right.
d = The value of the digit at position n or m.
b = The total number of digits in the number system.

Using this equation, binary, decimal, and hexadecimal sum as follows:

Binary
$$\sum_{n=0}^{\infty}(d_n \cdot 2)+\sum_{m=0}^{-\infty}(d_m \cdot 2^m)$$

Decimal
$$\sum_{n=0}^{\infty}(d_n \cdot 10^n)+\sum_{m=0}^{-\infty}(d_m \cdot 10^m)$$

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Hexadecimal
$$\sum_{n=0}^{\infty}(d_n \cdot 16^n)+\sum_{m=0}^{-\infty}(d_m \cdot 16^m)$$

Normally we only deal with non-fractional values, yet I have included the summation for fractional values to help show the generality of these equations.

Examples of how these equations work are given below:

BINARY

1	0	1	0	0	1	1	1
n = 7	6	5	4	3	2	1	0

$= 1\cdot 2^7+0\cdot 2^6+1\cdot 2^5+0\cdot 2^4+0\cdot 2^3+1\cdot 2^2+1\cdot 2^1+1\cdot 2^0$
$= 128+0+32+0+0+4+2+1$
$= 167$ (in decimal)

DECIMAL

4	4	1	0	6	9
n = 5	4	3	2	1	0

$= 4\cdot 10^5+4\cdot 10^4+1\cdot 10^3+0\cdot 10^2+6\cdot 10^1+9\cdot 10^0$
$= 400000+40000+1000+0+60+9$
$= 441069$ (in decimal)

HEXADECIMAL

E	4	1	A	D	7
n = 5	4	3	2	1	0

$= 14\cdot 16^5+4\cdot 16^4+1\cdot 16^3+10\cdot 16^2+13\cdot 16^1+7\cdot 16^0$
$= 14680064+262144+4096+2560+208+7$
$= 14949079$ (in decimal)

One last helpful tip about binary and hexadecimal is the ease of converting from one to the other. Since one hexadecimal digit has the same value as a four digit binary number, we can easily interchange the patterns of '0's and '1's in a binary number with hexadecimal numbers through substitution. To help, just look at the chart on the previous page.

BINARY HEXADECIMAL

	A	7	0	D
	1 0 1 0	0 1 1 1	0 0 0 0	1 1 0 1
n =	15 14 13 12	11 10 9 8	7 6 5 4	3 2 1 0

$=$ A 7 0 D (n = 3 2 1 0)

	0 0 0 1	1 1 1 1	1 0 1 0	1 0 0 0
n =	15 14 13 12	11 10 9 8	7 6 5 4	3 2 1 0

$=$ 1 F A 8 (n = 3 2 1 0)

	0 0 0 1	0 0 1 0	0 1 0 0	1 0 0 0
n =	15 14 13 12	11 10 9 8	7 6 5 4	3 2 1 0

$=$ 1 2 4 8 (n = 3 2 1 0)